

Improved sparse reconstruction for fluorescence molecular tomography with Poisson noise modeling

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Abstract: We present a maximum-likelihood-expectation-maximization (MLEM)-based method that models Poisson noise for improved reconstruction in fluorescence molecular tomography with sparse fluorescence distribution. © 2018 The Author(s)

1. Introduction

Measuring changes in membrane potential provides a mechanism to quantify neurotransmitter effects in the brain. In an NIH BRAIN initiative project [1, 2], our research group is developing new voltage sensitive dyes (VSDs) that are can be safely used in humans and emit optical radiation at near infrared wavelengths in response to membrane potential changes [3]. These optical measurements can be acquired using an FMT system. Our objective is to reconstruct the VSD distribution using the measurements made by the FMT system.

The reconstruction problem in FMT is highly ill-posed. However, we can use the fact that typically the VSD distribution is sparse. This enables the use of sparse reconstruction methods. Implementation of these sparse reconstruction methods is often preceded by some sort of preconditioning of the system matrix in FMT. These preconditioning operations improve the orthogonality of the system matrix, which helps improve the performance of the sparse reconstruction [4, 5]. Several preconditioning approaches are based on singular value decomposition (SVD) and preconditioning is directly apply on the sensing matrix. When the signal-to-noise ratio is poor, the performance of these preconditioning approaches are poor. To address this issue, we propose a novel approach based on compensating for the noise in FMT images using a maximum likelihood expectation maximization (MLEM) technique.

2. Method

The forward model of FMT is described by the following matrix equation:

$$\Phi = \mathbf{G}\mathbf{x} + \mathbf{n}, \quad (1)$$

where Φ is the vector for detector measurements, \mathbf{x} is a vector describing the unknown fluorescence distribution, \mathbf{G} is the sensitivity matrix and \mathbf{n} is the noise vector. To perform the preconditioning, a truncated SVD (TSVD)-based approach is implemented as described in [5], which converts equation (1) to

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \Sigma_t^{-1}\mathbf{U}_t^T \mathbf{n}, \quad (2)$$

where $\mathbf{y} = \Sigma_t^{-1}\mathbf{U}_t^T \Phi$ and $\mathbf{A} = \mathbf{V}_t^T$. The term Σ_t denotes a $K \times K$ truncated singular value matrix that keeps only the K largest singular values of \mathbf{G} , \mathbf{U}_t is truncated left singular vector matrix of size $M \times K$ and \mathbf{V}_t is truncated right singular vector matrix of size $N \times K$, where M and N are number of measurements and number of unknown voxel values, respectively. For sparse reconstruction, the following constrained optimization problem is formulated:

$$\min_x \|\mathbf{x}\|_0 \quad \text{such that} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \leq \epsilon \quad (3)$$

This optimization problem is NP hard, but can be approximately solved using different methods. In this work, we use two methods: compressive sampling matching pursuit (CoSaMP) method, and fast iterative shrinkage thresholding algorithm (FISTA) [6, 7]. The purpose of preconditioning is to enforce orthogonality of sensing matrix. Choosing a large value of K implies keeping a large set of singular values, some of which might be very small. As a result, the reconstruction is likely to be less robust to noise.

To address this issue, we propose to input the results from the sparse reconstruction into another reconstruction algorithm that accurately models the noise in the FMT systems. For this purpose, we develop an MLEM-based reconstruction algorithm that accurately models the Poisson noise in FMT systems. This algorithm can be easily derived by

writing the likelihood of the measured FMT data, where the likelihood accounts for the noise, and then estimating the fluorescence distribution that maximizes this likelihood. Note that similar MLEM algorithms have been widely used for PET and SPECT imaging. Typically, it has been observed that applying MLEM to FMT reconstruction has slow convergence. In this context, several pixel values in the sparse reconstruction have zero value. As MLEM is a multiplicative fixed-point technique, we observed that having a sparse initial estimate accelerates the MLEM reconstruction.

3. Experiments and results

We conducted simulation experiments with different levels of noise on a digital mouse phantom [8]. An FMT system was simulated and a fluorescence target was placed within the mouse brain. The whole brain was divided into 2942 voxels. 38 sources and 38 detectors were placed around the mouse head uniformly. The data simulated with this system was reconstructed using the proposed method and methods with only preconditioning and sparse reconstruction. The root mean square error (RMSE) was used to quantify the error.

Fig. 1 (a) shows the cross sections of reconstructed images as the truncation number was varied. Fig. 1 (b) and (c) show RMSE as a function of the truncation number for different noise levels. From the RMSE curve, we observe that for a range of truncation number values, the RMSE is stable. This range narrows as the noise level increases. Outside this proper range, for small truncation numbers, the reconstruction image is blurry and image bias is high; For large truncation numbers, noise overwhelms the image, as shown in the cross section figures. However, when we implement MLEM, the effect of noise is substantially reduced, and the fluorescence image is accurately reconstructed. A reduction is clearly observed in the RMSE curves when applying the MLEM technique for both CoSaMP and FISTA methods for different noise levels.

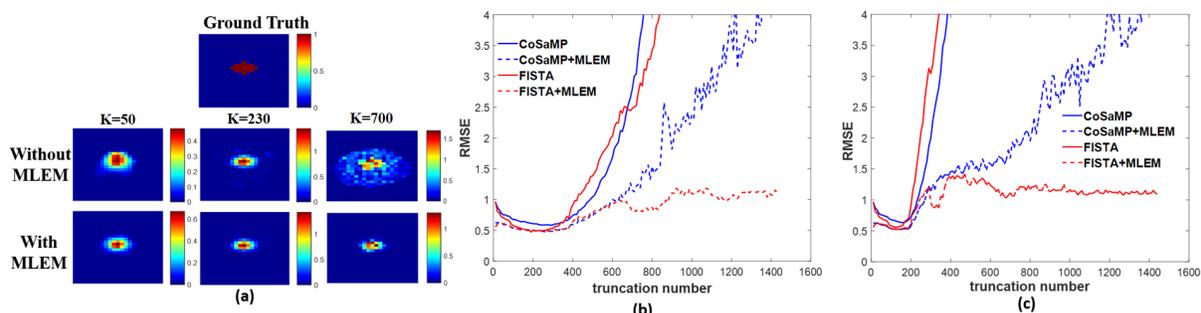


Fig. 1: (a) Cross sections of reconstruction results with different truncation numbers K . The images shown here are obtained with FISTA. Similar images can be obtained from CoSaMP. (b) RMSE curve for SNR=30dB. (c) RMSE curve for SNR=20dB.

4. Conclusion

An MLEM-based method was proposed in this paper to reconstruct fluorescence distribution for a FMT system. Simulation studies demonstrated that the method yields improved reconstruction accuracy. The method is more robust to noise after preconditioning compared to direct sparse reconstruction.

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